Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions:

There are EIGHT questions in all, out of which FIVE are to be attempted.

Questions no. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ENGLISH only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.
SECTION A

Q1. (a) What are the situations that depict the lack of control in $\bar{X}$ and $R$ charts? 8
(b) What are Military Standard Tables? Explain its uses in Statistical Quality Control theory? 8
(c) Explain the concepts of Type I censoring and Type II censoring. Describe the situation of them arising either by design or due to experimental circumstances. 8
(d) State the duality theorem in linear programming problem. Write the dual of the following primal problem: 8
Minimize $z = 2x_1 + 3x_2 + 4x_3$
subject to $2x_1 + 3x_2 + 5x_3 \geq 2$
$3x_1 + x_2 + 7x_3 = 3$
$x_1 + 4x_2 + 6x_3 \leq 5$
$x_1, x_2 \geq 0, x_3$ unrestricted in sign.
(e) Describe different features of a transition probability matrix in reference to a Markov chain. Given the following transition matrix of a Markov chain with three states 1, 2 and 3 and with initial probability distribution $\pi_0 = [0.7, 0.2, 0.1]$, find the value of
$$P[X_3 = 2, X_2 = 3, X_1 = 3]: \begin{bmatrix} 0.10 & 0.50 & 0.40 \\ 0.60 & 0.20 & 0.20 \\ 0.30 & 0.40 & 0.30 \end{bmatrix}$$

Q2. (a) Explain the meanings of (i) basic solutions, and (ii) feasible solutions in a linear programming problem with $m$ conditions and $n$ variables. Using simplex method, solve the following linear programming problem: 15
Maximize $z = 3x_1 + 2x_2 + 5x_3$
subject to $x_1 + 2x_2 + x_3 \leq 430$
$3x_1 + 2x_2 \leq 460$
$x_1 + 4x_2 \leq 420$
$x_1, x_2, x_3 \geq 0.$
(b) What are the different control charts for attributes used in Industrial Inspection of manufactured units? Also calculate the control chart for the number of defects and comment whether the process is under control or not based on the following data:

<table>
<thead>
<tr>
<th>Piece No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of defects</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>8</td>
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(c) Describe classification of states in a Markov chain. What is an n-step transition probability? Prove the following Chapman-Kolmogorov equation for transition probabilities:

\[ p_{ij}^{(n+1)} = \sum_k p_{jk}^{(n)} p_{kj}, \]

where symbols have their usual meanings.

Q3. (a) Define the terms of Reliability function and Failure rate function of a random variable denoting life time of a component. Establish the relation between them if any exist. Also prove that

\[ \int_0^\infty h(t) \, dt = \infty \]

where \( h(t) \) is the failure rate function.

(b) A food company puts mango juice into cans advertised as containing 200 ml of the juice. Quantity of the cans after filling for 10 samples of 4 cans each are taken by a random method at an interval of 60 minutes. Following presented below are the excess over 200 ml in each can. Construct an \( \bar{X} \)-chart to control the volume of mango juice for filling.

(\( A_2 \) for \( n = 4 \) is 0.729).

<table>
<thead>
<tr>
<th>Sample No.</th>
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<th>Can 3</th>
<th>Can 4</th>
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</table>
(c) What do you mean by an assignment problem (AP)? Describe the steps of the method for solving an AP.

A car rental service has a surplus of one car in each of the cities 1, 2, 3, 4, 5 and 6 and a deficit of one car in each of the cities 7, 8, 9, 10, 11 and 12. The distance (in kilometres) between cities with a surplus and cities with a deficit are shown in the matrix below. Work out an optimal assignment of surplus cars and the corresponding total distance to be travelled.

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Q4. (a) What is a multi-channel queueing problem? Deduce difference-differential equations for the (M/M/K) : (∞/FIFO) queueing system and obtain the steady-state solution for the system size.

(b) Develop hazard functions when the life pattern of a system was described by

(i) Exponential,
(ii) Weibull, and
(iii) Lognormal distribution.

(c) Describe a two-person zero sum game and in this context explain the terms (i) pay-off matrix, (ii) saddle point, and (iii) mixed strategies.

Let \( f(i, j) \) be a real-valued function and be defined whenever \( i \in A, j \in B \). Suppose that both \( \max_i \min_j f(i, j) \) and \( \min_j \max_i f(i, j) \) exist. Then prove that a necessary and sufficient condition that \( \max_i \min_j f(i, j) = \min_j \max_i f(i, j) \) is that the function \( f \) possesses a saddle point.
SECTION B

Q5.  (a) What are the components of time series data? Explain least square method of fitting a trend line.

(b) What is meant by Identification problem in simultaneous equation models? Distinguish between exactly identified, over identified and unidentified.

(c) With usual notations, explain abridged life table columns and establish the relationship between Age Specific Death Rate \( nM_x \) and Life Table Death Rate \( nq_x \).

(d) Explain the method of collecting demographic data using the method of registration, stating its uses and limitations.

(e) What are T-scores and standard scores? Mention the uses of T-scores and compare it with standard scores.

Q6.  (a) Explain time reversal and factor reversal tests. Show that the Marshall-Edgeworth index number lies in between Laspeyres' and Paasche's index numbers.

(b) What is Heteroscedasticity? Explain the following methods of detecting heteroscedasticity: (i) Graphical method, (ii) Park test, and (iii) Glejser test.

(c) Explain the need of using standardized death rates. Also describe the method of computing standardized death rate using indirect method.

Q7.  (a) Explain the role of CSO and NSSO, the official statistical organisations in India, in collecting statistics.

(b) Define Total Fertility Rate (TFR) and Net Reproduction Rate (NRR). Explain their importance in context of population growth.

(c) Explain different phases of Box-Jenkins methodology for time series data analysis using flow chart.
Q8. (a) Define and distinguish between stationary and stable populations. Explain Gompertz population growth model and state your comment.

(b) What is Multicollinearity? Give indicators which help in detecting multicollinearity. Also explain any two remedial measures to minimise multicollinearity.

(c) Explain the concepts of reliability and validity of scores in educational and psychological experiments. Mention the effect of lengthening a test upon reliability and validity. Test A has a reliability coefficient of 0.70 and a correlation of 0.40 with the criterion p. What would be the correlation of test A with the same criterion, if the test were tripled in length?