STATISTICS
Paper – III

General Instructions
This Question-Cum-Answer (QCA) Booklet contains 72+5 pages. Immediately on receipt of the Booklet, please check that this QCA Booklet does not have any misprint or torn or missing pages or items, etc. If so, get it replaced by a fresh QCA Booklet.
Candidates must read the instructions on this page and the following pages carefully before attempting the paper. Candidates should attempt all questions strictly in accordance with the specified instructions and in the space prescribed under each question in the Booklet. Any answer written outside the space allotted may not be given credit.
Question Paper in detachable form is available at the end of the QCA Booklet and can be removed and taken by the candidates after conclusion of the exam.

(To be filled by the Candidate)

Anukramik / Roll No. .................................................................
Pariksha ka Naam / Name of Examination .........................................

Vichar/Prashn-Patra / Subject/Paper
(Parvaeshak Durva Bhara Jaha)
(To be filled by Supervisor)

Upanishadit Sd.
Attendance No.

1011948

[Upavishak Kram Sa. Sarani Me Ya Prashn-Sah-Utar-Pustika Me
Anuvrat Nahi Bhara Jaha]
[Above Serial No. should not be written in the Table
or anywhere else in the Question-Cum-Answer Booklet]

Vishishak ke Hmasakar
Invigilator's Signature

ZLX-T-STS
Candidates should read the undermentioned instructions carefully. Violation of any of the instructions may lead to penalty.

DON'TS:

1. Do not write your Name or Roll number or Serial No. of Question-Cum-Answer-Booklet anywhere inside this Booklet. Do not sign the “Letter Writing” questions, if set in any paper by name, nor append your roll number to it.

2. Do not write anything other than the actual answers to the questions anywhere inside your Question Cum Answer-Booklet.

3. Do not tear off any leaves from your Question-Cum-Answer-Booklet. If you find any page missing, do not fail to notify the Supervisor/Invigilator.

4. Do not write anything on the Question Paper available in detachable form or admission certificate. Write answers at the specified space only.

5. Do not leave behind your Question-Cum-Answer-Booklet on your table unattended. It should be handed over to the Invigilator after conclusion of the exam.
व्याख्यादें योग्य बातें:

1. मुख्य पृष्ठ पर दिए गए अनुदेशों तथा इस पुस्तिका के अनुसार पृष्ठ पर इस प्रश्न-पत्र के लिए दिए गए विशेष अनुदेशों को ध्यानपूर्वक पढ़े और इनका काफी से अनुपालन करें।

2. प्रश्न-सह-उत्तर-पुस्तिका के मुख्य पृष्ठ पर दिए गए स्थान में अपना अनुक्रमक तथा अन्य विवरण लिखें।

3. स्वायत्त रूप में लिखें और लिखावट की सुसंगठन रखें। आरेख, चित्र, इत्यादि को बनाने के लिए पेस्ल का प्रयोग कर सकते हैं।

4. एक कार्य (लेखनीय या ज्यादातर) के लिए, इस पुस्तिका के अन्त में दिए गए खाली पृष्ठों का प्रयोग किया जाना चाहिए। एक कार्य को बाद में पूर्ण तरह से काट दें।

5. यदि आप अपने किसी उत्तर को रद्द करना चाहें तो उसे पेन से काट दें और उस पर “रद्द” लिख दें, अन्यथा उसका मूल्यांकन किया जा सकता है।

6. परीक्षा भवन छोड़ने से पहले अपनी प्रश्न-सह-उत्तर-पुस्तिका व्यक्तिगत रूप से निरीक्षक को सौंप दें।

DO’S:

1. Read the instructions on the cover page and the specific instructions to this Question Paper mentioned on the next page of this Booklet carefully and strictly follow them.

2. Write your Roll number and other particulars, in the space provided on the cover page of the Question-Cum-Answer-Booklet.

3. Write legibly and neatly in ink. Pencil may be used for drawing diagrams, sketches, etc.

4. For rough work, blank pages provided at the end of this booklet should be used. The rough work should be crossed through afterwards.

5. If you wish to cancel any work, draw your pen through it or write “Cancelled” across it, otherwise it may be valued.

6. Hand over your Question-Cum-Answer-Booklet personally to the invigilator before leaving the examination hall.

<table>
<thead>
<tr>
<th>प्रश्न सं. Questions No.</th>
<th>सुरुआती पृष्ठ सं. Starting Page No.</th>
<th>अंक Marks</th>
<th>खण्ड योग Section Total</th>
<th>परीक्षक के हस्ताक्षर Signature of Examiner</th>
</tr>
</thead>
<tbody>
<tr>
<td>खण्ड A Section A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>खण्ड B Section B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>61</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

सकल योग/Grand Total

(ZLX-T-STS)
Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions:

There are **EIGHT** questions divided under **TWO** sections.

Candidate has to attempt **FIVE** questions in all.

Both the **TWO** questions in Section A are **compulsory**.

Out of the **SIX** questions in Section B, any **THREE** questions are to be attempted.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.
Both the questions are compulsory.

For SRSWOR (N, n), show that the sample proportion \( p \) is unbiased for the population proportion \( P \). Also derive the sampling variance of this estimator.
Q1(b) What is the problem in estimating a linear regression model in presence of multicollinearity? How is multicollinearity detected? Explain how ridge estimation tackles this issue.
Consider the MA(1) process \( X_n = e_n + \beta e_{n-1} \), where \( e_n \sim N(0, 1) \).

For a data set it is noted that autocovariances are \( \hat{\gamma}_0 = 1 \) and \( \hat{\gamma}_1 = -0.25 \).

(i) Estimate \( \beta \). Which value of the estimate do you think we should choose and why?

(ii) What problem do we have if \( \hat{\gamma}_1 = -0.5 \)? How would the variance of the error have affected the change?
Q2. (a) Given below are the figures on production (in thousand metric tons) of a cooperative sugar factory:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>77</td>
<td>88</td>
<td>84</td>
<td>85</td>
<td>91</td>
<td>98</td>
<td>90</td>
</tr>
</tbody>
</table>

(i) Fit a linear trend by least squares method. Tabulate the trend values.

(ii) Compute the monthly estimated increase in production during the period.
SPECIMEN
Q2(b) If, in every stratum, the simple estimator $\bar{y}_h$ is unbiased, then show that

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$$

is unbiased for population mean $\bar{y}$, where $W_h$ is the proportion of population units in the strata and $L$ denotes the total number of strata in the population.

Derive the sampling variance of $\bar{y}_{st}$ and state how you would unbiasedly estimate the same.
In the context of a finitely distributed lag model, discuss the problem of OLS estimation and suggest how to obtain good (consistent) estimates of the parameters in such a model by bringing in some restrictions on lag weights.
SECTION B

Answer any three questions of the six questions given below.

Q3. (a) Explain and illustrate the following:
(i) Two-stage sampling
(ii) Two-phase sampling
Pinpoint the difference between the two types of sampling schemes.
Q3(c) Discuss the following allocations of the sample size in stratified random sampling:

(i) Proportional allocation

(ii) Neyman allocation

(iii) Optimum allocation with a linear cost function

Explain the practical implications of these methods.
Q4. (a) Discuss Koyck approach to an infinitely distributed lag model and obtain the mean lag for Koyck’s model. What are the basic features of Koyck’s transformed model?
Q4(b) For the following linear regression model
\[ Y_i = \beta_1 + \beta_2 X_i + u_i \quad i = 1, ..., n \]
\[ \text{E}(u_i) = 0, \quad \text{Cov}(u_i, u_j) = 0 \quad i \neq j, \quad \text{E}(u_i^2) = \sigma_i^2, \]

obtain the OLS estimates of \( \beta_1 \) and \( \beta_2 \) when \( \sigma_i^2, \ i = 1, ..., n \) are known. Discuss what steps could be taken when \( \sigma_i^2, \ i = 1, ..., n \) are unknown. Revise the least squares estimates of \( \beta_1 \) and \( \beta_2 \) when \( \sigma_i^2 = \sigma^2 \text{E}^2(Y_i) \) where \( \sigma^2 \) is unknown.
SPECIMEN
Q4(c) Discuss the problem of estimating parameters by OLS in the presence of serial correlation in the following model:

\[ Y_t = \beta_1 + \beta_2 X_t + u_t \]

\[ u_t = \rho u_{t-1} + \varepsilon_t, \quad -1 < \rho < 1, \quad \rho \text{ is known.} \]

\[ E(\varepsilon_t) = 0, \quad V(\varepsilon_t) = \sigma^2, \quad \text{Cov}(\varepsilon_t, \varepsilon_{t+s}) = 0 \quad s \neq 0. \]

Propose suitable estimates of \( \beta_1 \) and \( \beta_2 \). Also calculate the variance of the estimate of \( \beta_2 \). How can this estimate be modified when \( \rho \) is unknown?
Demand and supply functions of a certain commodity are respectively

\[ x_d = 240 + 10 \frac{dp}{dt} - 4p \text{ kg per month; } \]

\[ x_s = 100 \frac{dp}{dt} + 6p - 60 \text{ kg per month, } \]

where \( p \) is price of the commodity at time \( t \).

Find the time path of \( p \) for dynamic equilibrium if the initial price is to be ₹ 72 per kg.
Q5(b) Explain briefly the methods of computing price index numbers

(i) by simple average of price relatives;
(ii) by simple aggregate of prices; and
(iii) by weighted aggregate of prices.
Q5(c) Discuss the different forms of the Engel curve that are usually employed for fitting to family-budget data. In such fitting, how would you tackle the following complications?

(i) Household expenditure on a particular item depends, besides depending on income, on the number of persons per family.

(ii) Consumption of families of the same size differs because of varying age and sex consumption.
SPECIMEN

SPECIMEN
Q6. (a) Give an illustration for linear systematic sampling. Show that, under this method, a positive correlation between units in the same sample inflates the sampling variance of the estimator of population total.
Q6(b) Consider a population of N = 6 units with values 1, 2, 3, 4, 5 and 6.

(i) Write down all possible samples of size 2 drawn by SRSWOR scheme.
    Verify that the sample mean is unbiased for the population mean.

(ii) Also compute the sampling variance of the sample mean.
Q6(c) Explain the ratio method of estimation for estimating a population total. Show that it is generally biased. Evaluate the mean squared error of the estimator to the first order of approximation. Assume SRSWOR of n units from the population.
Q7.(a) Using standard notations, briefly explain the instrumental variable technique in the context of estimating the coefficients in a linear regression model. State the situations when this technique is applicable.
Q7(b) State the rank and order conditions for identifiability of parameters in a system of structural equations. Which one of these two conditions is sufficient for identifiability? Establish this condition mathematically.
Q7(c) Discuss the estimation of parameters of an equation appearing in a simultaneous equation system by Limited Information Maximum Likelihood method. State whether the estimator (if it exists) is unique. (An outline of the approach is adequate)
Show that the relationship \( X_t = 0.7X_{t-1} + 0.3X_{t-2} + \varepsilon_t + 0.7\varepsilon_{t-1} \) (where \( \varepsilon_t \) denotes white noise) defines ARIMA(1, 1, 1) model.
Q8(b) What do you understand by the seasonal variations in a time series? Give an example. Explain the method of link relatives of computing the seasonal indices.
Q8(c) Define correlogram.
For an infinite series generated by the average of a random component with equal weights, show that the correlogram is

$$
\rho_k = \begin{cases} 
1 - \frac{k}{m} & \text{for } k \leq m \\
0 & \text{for } k > m
\end{cases}
$$