GEO-PHYSICS

PAPER—III

Time Allowed : Three Hours
Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are TEN questions divided in TWO sections.
Candidate has to attempt SIX questions in all.
Question Nos. 1 and 6 are compulsory. Out of the remaining EIGHT questions, FOUR questions are to be attempted choosing TWO from each section.
The number of marks carried by a question/part is indicated against it.
Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.
Unless otherwise mentioned, symbols and notations have their usual standard meanings.
Assume suitable data, if necessary and indicate the same clearly.
Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.
Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.
Answers must be written in ENGLISH only.

Constants which may be needed:

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kepler's constant</td>
<td>$3.986004418 \times 10^5$ km$^3$-s$^{-2}$</td>
</tr>
<tr>
<td>Mean radius of the earth</td>
<td>6378 km</td>
</tr>
<tr>
<td>Mass of electron ($m_e$)</td>
<td>$9.11 \times 10^{-31}$ kg</td>
</tr>
<tr>
<td>Charge of electron ($e$)</td>
<td>$1.602 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Planck's constant ($h$)</td>
<td>$6.62 \times 10^{-34}$ J·s</td>
</tr>
<tr>
<td>Boltzmann's constant ($k$)</td>
<td>$1.38 \times 10^{-23}$ J/K</td>
</tr>
<tr>
<td>Permittivity of free space ($\varepsilon_0$)</td>
<td>$8.854 \times 10^{-12}$ F·m$^{-1}$</td>
</tr>
</tbody>
</table>
SECTION—A

1. (a) Compute the geoid height anomaly resulting from an isostatic density distribution (Airy-compensated) of ocean basin 6 km deep.

(b) A 30 Hz seismic signal reflects from the base of a 2 km thick layer with velocity \( v = 4 \text{ km/s} \). What is the minimum size of horizontal feature that could be detected?

(c) Draw schematically the gravity and magnetic anomalies over a given two-dimensional body (see the figure below). Explain the salient features on the difference between the two anomalies.

Direction of magnetization in the body is marked by an arrow.

(d) Calculate the angles of refraction for P-wave and S-wave for an incident P-wave undergoing mode conversion.

(e) Sketch the sequence \( x(n) = 0 \cdot 5 \delta(n + 1) + 3 \delta(n - 1) + 2 \delta(n - 2) \).

(f) An analog signal is given as follows:

\[ x_a(t) = 3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t \]

Find the Nyquist rate.

(g) Write briefly the different types of remote sensing methods and explain the importance of electromagnetic radiation in remote sensing studies.

(h) Enumerate the causes and consequences of excess water drawn in coastal aquifers.

2. (a) (i) Calculate the normal-incidence P-wave reflection and transmission coefficients for an interface separating two different media as shown below:

\[
\text{Basalt (6.5 km/s, 2.6 g/cc)} \quad \text{Interface} \quad \text{Granite (7.2 km/s, 3.0 g/cc)}
\]

(ii) What crustal density is required to explain the 5-km high mountain range? Assume density of crust = 2800 kg/m\(^3\), density of mantle = 3200 kg/m\(^3\) and crustal thickness = 40 km.
(b) Illustrate with neat sketches the primary and secondary magnetic fields induced in a subsurface conductor, as in a horizontal loop electromagnetic induction method. Draw amplitudes and phases of primary and secondary fields. How is the phase shift between the two fields related to skin depth? 8+2=10

(c) Define gravitational potential due to a distribution of infinitesimal masses each at position \( r \).

Explain quantitatively with neat sketches the potential of a spherical shell of radius \( b \) at a point (i) outside \( r > b \) and (ii) inside \( r < b \) the spherical shell. 2+8=10

3. (a) Define convolution. Obtain the convolution of the sequence

\[
h(n) = \{1, 2, 3\} \\
x(n) = \{1, 2, 2, 1\}
\]

(b) (i) Compute the horizontal gravity gradient for gravity measurement at a local base station at 25° N latitude.

(ii) A radioactive parent \( (P) \) atom disintegrates to a stable daughter \( (D) \) atom at time \( t \). Express the elapsed time \( t \) in terms of \( P \) and \( D \). 5

(c) For a half-space overlain by seawater (see the figure below), draw schematically the ray paths and travel time curves for P-waves propagating into thick sediments with a velocity gradient at distances (i) less than and (ii) greater than critical distance.

\[
\begin{align*}
\text{Ocean surface} \\
\text{Seawater} & \quad \text{(Constant velocity)} \\
\text{Seafloor} \\
\text{Sediments} & \quad \text{(Constant velocity gradient)}
\end{align*}
\]

4. (a) (i) Write about the significance of sensors in remote sensing.

(ii) Write about the different types of remote sensing platforms.

(b) Differentiate between analog and digital filters.

(c) (i) Briefly write about the vertical distribution of groundwater and sketch the subsurface water distribution.

(ii) Write about the factors that influence specific yield.
5. (a) Find the inverse z-transform of

\[ X(z) = \frac{z^{\frac{3}{2}}}{(z - \frac{1}{3})(z - \frac{1}{2})}; \text{ROC : } |z| > \frac{1}{2} \]

(b) (i) Write about the significance of aerial photography in remote sensing applications.

(ii) Briefly write about the Darcy's law and its application to understand groundwater flow in subsurface.

(c) (i) Draw the relationship between amplitude/phase of a secondary electromagnetic field and the product of conductivity and frequency, as used in quadrature EM method.

Highlight the problem with this method in resolving good and poor conductors, and how to overcome it.

(ii) Mention the disadvantages of using air gun source and how to overcome it, as in marine seismic survey.

Compare schematically the source signatures of a single air gun and an array of air guns.

**SECTION—B**

6. (a) Determine the Miller indices of the directions which are common to (101) and (111) planes in a cubic unit cell.

(b) How do entropy and specific heat vary with temperature for a superconductor?

(c) A three-level laser emits laser light at a wavelength of 550 nm. If the optical pumping mechanism is shut off, what will be the ratio of the population of the upper level (energy \( E_2 \)) to that of the lower level (energy \( E_1 \))? Assume that \( T = 300 \) K.

(d) Imagine that we chop a continuous laser beam \( (\lambda = 623.8 \text{ nm}) \) into 0.1 ns pulses using some sort of shutter. Compute the resultant linewidth, bandwidth and coherent length.

(e) Differentiate between active satellite and passive satellite.

(f) If a radar of 1 MW peak power and antenna gain of 1000 irradiates a 1 m² target with a 10 microsecond pulse at 1000 km range, what energy density arrives back at the radar and in how much elapsed time?
(g) Use uncertainty principle to estimate (i) the ground state radius of hydrogen atom and (ii) the ground state energy of hydrogen atom.

(h) Show that the phase velocity of a non-relativistic free particle is half of the group velocity.

7. (a) Derive Bragg’s law of X-ray diffraction in crystals. Give an account of powder method of crystal structure analysis.

(b) Explain three- and four-level pumping schemes used in lasing action. Which one is better and why?

(c) (i) Convert 11.1101 from binary system to decimal system.
(ii) Implement the Boolean function \( X = AB + \bar{A}C \) with NAND gates.

8. (a) The electron and hole mobilities in a silicon sample are 0.135 \( \text{m}^2/\text{V-s} \) and 0.048 \( \text{m}^2/\text{V-s} \). Determine the conductivity of intrinsic Si at 300 K if the intrinsic carrier concentration is \( 1.5 \times 10^{16} \) atoms/\( \text{m}^3 \). The sample is then doped with \( 10^{23} \) phosphorus atoms/\( \text{m}^3 \). Determine the equilibrium hole concentration, conductivity and position of the Fermi level relative to the intrinsic level.

(b) Explain how optical cavities are useful in laser operation. What are different losses in optical cavities?

(c) Calculate \([J_x^2, J_y]\), \([J_z^2, J_y]\) and \([J^2, J_y]\). Then show that
\[
\langle J, m | J_x^2 | J, m \rangle = \langle J, m | J_y^2 | J, m \rangle
\]

9. (a) Why is the field effect transistor called a unipolar transistor? Explain the principle of operation of an \( n \)-channel JFET.

As \( V_{GS} \) is changed from \(-1 \text{ V}\) to \(-1.5 \text{ V}\), keeping \( V_{DS} \) constant, \( I_D \) of a FET drops from 7 mA to 5 mA. What is the transconductance of the FET?

(b) How does a semiconductor laser differ from other lasers?

Determine the temperature at which the rates of spontaneous and stimulated emissions are equal. Given \( \lambda = 500 \text{ nm} \).

(c) Use the variational method to estimate the ground state energy of hydrogen atom.
10. (a) (i) Gold has the same structure as copper. The velocity of sound in gold is 2100 m/s and that in copper is 3800 m/s. If the Debye temperature of copper is 348 K, determine the Debye temperature of gold. The densities of gold and copper are $1.93 \times 10^4 \text{ kg/m}^3$ and $8960 \text{ kg/m}^3$, and their atomic weights are 197.0 a.m.u. and 63.54 a.m.u., respectively.

(ii) An n-p-n transistor with $\alpha = 0.98$ is operated in the CB configuration. If the emitter current is 3 mA and the reverse saturation current is $I_{CO} = 10 \mu A$, what are the base current and the collector current?

(b) (i) A satellite is moving in an elliptical orbit with the major axis equal to 42000 km. If the perigee distance is 8000 km, find the apogee and the orbit eccentricity.

(ii) Explain the role of an earth station in overall satellite communication setup.

(c) A particle of energy $E$ is incident on a potential step whose potential function is given by

\[ V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases} \]

Given that $E > V_0$.

Write down the Schrödinger wave equation for the particle and find its solution. Also, find the reflection and transmission coefficients.