QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ENGLISH only.
Useful Constants

Mass of proton = $1.673 \times 10^{-27}$ kg
Mass of neutron = $1.675 \times 10^{-27}$ kg
Mass of electron = $9.11 \times 10^{-31}$ kg
Planck constant = $6.626 \times 10^{-34}$ J s
Boltzmann constant = $1.380 \times 10^{-23}$ J K$^{-1}$
Bohr magneton ($\mu_B$) = $9.273 \times 10^{-24}$ A m$^2$
Nuclear magneton ($\mu_N$) = $5.051 \times 10^{-27}$ J T$^{-1}$ (A m$^2$)
Electronic charge = $1.602 \times 10^{-19}$ C
Atomic mass unit (u) = $1.660 \times 10^{-27}$ kg
= 931 MeV

g_s = 5.5855 \mu_N
m(p) = 1.00727 u
m(n) = 1.00866 u
m($^3_6$C) = 12.00000 u
m($^2_1$H) = 2.014022 u
m($^1_8$O) = 15.999 u
h = $1.05 \times 10^{-34}$ J s
m($^4_2$He) = 4.002603 u
m($^{87}_{36}$Sr) = 86.908893 u
m($^3_1$H) = 3.0160500 u
$\varepsilon_0$ = 8.854 $\times 10^{-12}$ F/m
$hc$ = 197 eV nm
SECTION—A

1. (a) An electron and a photon each has a wavelength of 2 Å. Calculate (i) their momenta and (ii) the ratio of their kinetic energies.

(b) Show that the phase velocity \( v_p \) for a particle with rest mass \( m_0 \) is always greater than the velocity of light and that \( v_p \) is a function of wavelength.

(c) For Pauli’s matrices, prove that (i) \( [\sigma_x, \sigma_y] = 2i\sigma_z \) and (ii) \( \sigma_x\sigma_y\sigma_z = i \).

(d) The first rotational line of \( ^{12}\text{C}^{16}\text{O} \) is observed at 3·84235 cm\(^{-1} \) and that of \( ^{13}\text{C}^{16}\text{O} \) at 3·67337 cm\(^{-1} \). Assuming the mass of \( ^{16}\text{O} \) to be 15·9949 u, calculate that of \( ^{13}\text{C} \).

(e) Draw the schematic diagram of a nuclear magnetic resonance (NMR) spectrometer. Discuss nuclear magnetic resonance imaging.

2. (a) An electron is confined in the ground state of a one-dimensional harmonic oscillator such that \( \Delta x = 10^{-10} \) m. Assuming \( \langle T \rangle = \langle V \rangle \), find the energy in eV required to excite it to the first excited state.

(b) In the \( |jm\rangle \) basis formed by the eigenkets of \( J^2 \) and \( J_z \), show that

\[
\langle jm\mid J_-J_+\mid jm\rangle = (j - m)(j + m + 1)\hbar^2
\]

where \( J_+ = J_x + iJ_y \) and \( J_- = J_x - iJ_y \).

(c) Write down the Franck-Condon principle and explain how it accounts for the intensities of spectral lines in vibrational-electronic spectra with suitable illustrations.

3. (a) If the state of the hydrogen atom is \( 2p \) state, calculate the energy levels of the spin-orbit interaction Hamiltonian \( A \vec{L} \cdot \vec{S} \), where \( A \) is a constant.

(b) Consider a particle whose wave function is given by \( \psi(x) = Ae^{-\alpha x^2} \).

(i) What is the value of \( A \) if this wave function is normalized?

(ii) Calculate the expectation value of \( x \) for this particle.

(c) What is Lamb shift? Why does the study of Lamb shift enjoy so much attention?

4. (a) If the Hamiltonian of a system be \( H = (p_x^2 / 2m) + V(x) \), obtain the value of the commutator \([x, H]\). Hence find the uncertainty product \( (\Delta x)(\Delta H) \).
(b) Discuss the anomalous magnetic splitting of sodium D-lines with necessary illustration (no deduction is required). Identify the polarization of each component line. What condition needs to be satisfied for this splitting to take place?

(c) Write down the characteristics of Raman spectral lines. Explain why Stokes lines are more intense than anti-Stokes lines. With exciting line at 2536 Å, a Raman line for a sample is observed at 2612 Å. Calculate the Raman shift in wave number.

SECTION—B

5. (a) What are the properties of the particles made up of the following quarks? Also write the names of particles :

(i) u̅d
(ii) u̅d
(iii) d̅s
(iv) u̅s

(b) What do you understand by the reproduction factor of a fission reactor? What is the maximum value of the reproduction factor in a uranium fission reaction?

(c) In the following nuclear reactions, what particles are possible for the unknown particle X?

(i) \( \pi^- + p \rightarrow K^0 + X \)
(ii) \( p + p \rightarrow \pi^+ + n + \Lambda^0 + X \)

(d) A superconducting material has a critical temperature of 3.7 K and a magnetic field of 0.0306 tesla at 0 K. Find the critical field at 2 K.

(e) Obtain the Boolean expression for the output \( Y \) in the given logic circuit:

![Logic Circuit Diagram](image)

Simplify the expression and show that the circuit is equivalent to an AND gate with inputs \( A \) and \( B \).

6. (a) On the basis of liquid-drop model, write the semi-empirical mass formula for the total binding energy. Also explain their each term.

(b) Explain the Meissner effect in superconductivity. Obtain an expression for London penetration depth of magnetic field for a superconductor.
(c) An n-channel silicon JFET has donor concentration of $2 \times 10^{21} / \text{m}^3$ and a channel width of 4 $\mu$m. If the dielectric constant of silicon is 12, find the pinch-off voltage. If the FET operates with a gate-source voltage of $-2$ V, what is the saturation voltage $V_{D_{sat}}$?

7. (a) Calculate the decay rate of $^{14}$C per gram of carbon in a living organism, assuming the ratio $^{14}$C/$^{12}$C = $1.35 \times 10^{-12}$. The half-life of $^{14}$C is 5730 years.

(b) By applying the conservation laws, state about the following decay equations, whether they are allowed or not allowed:

(i) $\Sigma^+ \rightarrow p + \pi^0$
(ii) $\Sigma^0 \rightarrow \Lambda^0 + \gamma$
(iii) $\Lambda^0 \rightarrow n + \pi^0$

(c) Explain the use of an op-amp as a differentiator and an integrator using proper circuit diagram. Write the condition to work as good differentiator and integrator.

8. (a) Explain the salient features of nuclear shell model. What are the magic numbers? Write the experimental facts in support of shell model of nuclei.

(b) A transistor having $\alpha = 0.975$ and a reverse saturation current $I_{CO} = 10 \mu A$ is operated in CE configuration. What is $\beta$ for this configuration? If the base current is 250 $\mu A$, calculate the emitter current and the collector current.

(c) Describe briefly the construction of an enhancement type MOSFET using a $p$-type silicon bar. Point out the structural difference between enhancement and depletion forms of the MOSFET.